TESTS OF SU(3) SYMMETRY-BREAKING FOR BARYONIC BETA TRANSITIONS IN THE STANDARD MODEL *

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Abstract

It is generally assumed that deviations from flavor SU(3) symmetry arise entirely from quark mass-differences, reflected in the mass splittings between strange and nonstrange members of the same SU(3) multiplet. Under this assumption, a parametrization is proposed which expresses the ratios of Gamow-Teller to Fermi matrix elements for nucleon and hyperon beta decays entirely in terms of two SU(3)-invariant coupling constants F and D and two parameters γ and δ representing SU(3) breaking effects. Therefore, in principle, measurement of this ratio for any four beta-transitions should yield all four parameters. Available data do not show any evidence for SU(3) breaking. Improved measurements, also for transitions not previously measured, would provide more stringent tests.

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Introduction

In the Standard Model, based on gauge-interactions of leptons and quarks, if one ignores the influence of more massive quarks belonging to the third family, charge-changing weak interactions of strange and non-strange hadrons are induced by the current $\bar{u}\gamma_{\alpha}(1+\gamma_{5})d_{c}$ with

$$d_c = d \cdot \cos\theta + s \cdot \sin\theta,\tag{1}$$

where the particle symbols represent the quark fields and the single parameter [1,2] θ characterizes the intrinsic difference in strength between the weak couplings of strangeness-changing and strangeness-conserving currents. Had d and s quarks been degenerate in mass, d_c and its orthogonal partner s_c would have been equally valid choices for the basis states (d, s) of charge-(-1/3) quarks, and there would have been no need to introduce θ at all. From this viewpoint, the charged weak current transforms like a SU(2) generator, as it would be required to if it solely induced $d \leftrightarrow u$ transitions. However, for reasons not yet understood, it operates in a SU(2) subspace [of the SU(3) space spanned by (u, d, s) states] which is skewed with respect to the SU(2) space spanned by the (nearly) degenerate (u, d) states [3]. ΔS =1 weak interactions occur only because the direction in (flavor)- SU(3) space chosen by the charged weak current, lies outside the subspace which is orthogonal to the direction containing the more massive s quark.

The description of baryonic beta-transitions in this theory requires, in addition to the parameter θ mentioned above - which we shall take as already determined from comparison of strange and non-strange meson decays - an additional parameter to determine the relative contribution of two different ways to construct a SU(3)-octet matrix-element between the initial and final baryon octet states, even in the limit of exact SU(3) symmetry [2]. In the Standard Model, this parameter F/D would be determined by the quark structure of the baryon states which, regrettably, is only poorly known, basically because we do not yet have a reliable way to calculate hadronic strong-interaction effects. For this reason, empirical knowledge of this parameter is of great interest. The value deduced from global analyses of hyperon and nucleon beta-decays has been combined [4] with measurements of deep inelastic electron or muon scattering from polarized nucleon targets, to make inferences about the distribution of spin within the nucleon. Because the deduced distributions do not conform to theoretical expectations, various [5,6] proposals have been advanced to resolve this so-called "spin crisis". Among these is the suggestion [7] that the effects of SU(3) symmetry-breaking, arising from quark mass-differences, may be so large as to invalidate the values of F/Ddeduced from the analysis of hyperon beta-decays, which did not take into account such effects. The aim of this brief report is to present an analysis of baryonic beta-transitions, including effects of mass-differences, extending the earlier perturbative discussions [8,9], which appears to be more convenient for the present purpose. Applied to the available data, we find no evidence for significant departure from the SU(3)-symmetric results, in agreement with our results [10] from a purely perturbative analysis published earlier. Four relatively well-measured baryonic beta-transitions permit a unique determination of the parameters Fand D, and two additional parameters describing SU(3)-breaking effects, in our approach.

A clear test of our suggested method would be to see if *other* as yet poorly measured beta transitions of hyperons are well-described by the *same* set of four parameters. Section 2 outlines our method of analysis. The assumptions underlying our calculation are stated,

and the resulting formula for g_A/g_V for baryonic beta-transitions is worked out for the cases of interest. By applying this to the available data, we find the best values for F and D; two additional parameters, representing SU(3)-breaking effects in our method of analysis, are found to be not distinguishable from zero, within the reported errors of measurement. Our conclusions are summarized in Section 3.

2. Baryonic Beta-Transitions including SU(3) Breaking

We shall assume that beta-transitions between octet baryons are induced by vector and axial-vector charged weak currents, coupled to corresponding V-A currents of leptons. In the Standard Model, these currents involve transitions between the basic quarks, and therefore transform as (flavor)-SU(3) octet operators. Under the assumption that all deviations from flavor-SU(3) symmetry arise from differences between the masses of strange and non-strange quarks [3], the matrix-elements of the weak current will transform in the same way under charge-conjugation as they would in the symmetric limit. With the Cabibbo current:

$$j_W^{\mu} = \bar{q} \frac{\lambda_W}{2} \gamma^{\mu} (1 + \gamma_5) q \tag{2}$$

where

$$\lambda_W = (\lambda_1 + i\lambda_2)\cos\theta + (\lambda_4 + i\lambda_5)\sin\theta \tag{3}$$

and θ is Cabibbo's angle and the λ -matrices are defined in [1,11], the generalization of the assumption that "first-class" currents [12] are the only ones that occur, is

$$(G'P)J_{\alpha}(G'P)^{-1} = -J_{\alpha} \tag{4}$$

where $G' = C.e^{i\pi I_2'}$ with $I_2' = F_2\cos\theta + F_5\sin\theta$, as would be the case in the Standard Model. In the allowed approximation [13], which suffices to describe the existing data, the general form of the baryonic matrix-elements, linear in the weak current and in μ (defined below), with the stated transformation properties, is

$$M = a||\lambda_W \bar{\mathbf{B}}\mathbf{B}|| + b||\lambda_W \mathbf{B}\bar{\mathbf{B}}|| + c(||\lambda_W \mathbf{B}\mu\bar{\mathbf{B}}|| - ||\lambda_W \bar{\mathbf{B}}\mu\mathbf{B}||) + d(||\lambda_W \mathbf{B}|| \cdot ||\mu\bar{\mathbf{B}}|| + ||\lambda_W \bar{\mathbf{B}}|| \cdot ||\mu\mathbf{B}||)$$
(5)

where $||X|| \equiv \text{Tr}(X)$ and **B** is the matrix representing the baryon octet:

$$\mathbf{B} = \begin{pmatrix} \frac{1}{\sqrt{2}} \Sigma^0 + \frac{1}{\sqrt{6}} \Lambda^0 & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}} \Sigma^0 + \frac{1}{\sqrt{6}} \Lambda^0 & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}} \Lambda^0 \end{pmatrix}.$$
 (6)

In the SU(3) symmetric limit, the only possible SU(3)-octet matrix-elements between octet baryons are those given by the first two terms in Eq.(5). The effects of SU(3)-symmetry breaking are included in the terms containing

$$\mu = (I - \sqrt{3}\lambda_8)/3. \tag{7}$$

The use of the idempotent operator μ instead of λ_8 offers the advantage that higher powers are automatically included [14].

It is known [15,8] that the effects of SU(3)-breaking, represented here by the operator μ , do not appear in first order for the vector coupling, and we shall disregard them accordingly. If we extend to the Gell-Mann-Le'vy-Cabibbo current, Eq.(2), the notion of the Conserved Vector Current, then the general form of the vector (Fermi) matrix-element will be further constrained by the requirement b = -a in Eq. (5). With these assumptions, Eq.(5) yields, for the ratio r of Gamow- Teller [16] to Fermi matrix-elements for the four best-measured beta- transitions [11]:

$$r_1 = (-g_A/g_V)_{n \to p} = F + D + \gamma,$$
 (8a)

$$r_2 = (-g_A/g_V)_{\Lambda \to p} = F + D/3 + 2(\gamma + \delta)/3,$$
 (8b)

$$r_3 = (-g_A/g_V)_{\Sigma^- \to n} = F - D,$$
 (8c)

$$r_4 = (-g_A/g_V)_{\Xi^- \to \Lambda} = F - D/3 + 2(\gamma - \delta)/3.$$
 (8d)

It will be seen that these formulas require, in addition to the two SU(3) parameters F and D, called for in the symmetric limit, only two further parameters [17] γ and δ to describe the effects of SU(3)-breaking. In principle, all four parameters are uniquely determined by the four g_A/g_V ratios.

From these equations, we obtain the following relations between F, D, and the two additional parameters γ , and δ , required to describe SU(3) breaking, and the reported g_A/g_V ratios:

$$F = 2(r_1 + r_3) - \frac{3}{2}(r_2 + r_4), \tag{9a}$$

$$D = 2r_1 + r_3 - \frac{3}{2}(r_2 + r_4), \tag{9b}$$

$$\gamma = 3[(r_2 + r_4) - (r_1 + r_3)], \tag{9c}$$

$$\delta = \frac{1}{2}(3r_2 - 2r_1 - r_3). \tag{9d}$$

The Gell-Mann-Le'vy-Cabibbo hypothesis, - that the weak currents should transform as SU(3) octets, - would require that, in the absence of SU(3) breaking effects, each of the two parameters γ and δ should vanish. Inserting the current values [11] for the r_j (j = 1 - 4),

$$r_1 = 1.2670 \pm 0.0035,\tag{10a}$$

$$r_2 = 0.718 \pm 0.015,\tag{10b}$$

$$r_3 = -0.340 \pm 0.017,\tag{10c}$$

$$r_4 = 0.25 \pm 0.05,\tag{10d}$$

in Eqs.(9a)-(9d), we find

$$F = 0.402 \pm 0.085,\tag{11a}$$

$$D = 0.742 \pm 0.080,\tag{11b}$$

$$\gamma = 0.123 \pm 0.164,\tag{11c}$$

$$\delta = -0.020 \pm 0.024,\tag{11d}$$

which shows that, within the stated errors, there is no significant evidence for the SU(3)-

breaking effects suggested by some authors[7]. It will be seen that the parameter δ is determined with relatively high precision and imposes quite a strict limit on possible SU(3) breaking. Because of the large error associated with the reported value (10d), for the ratio of g_A/g_V for the $\Xi^- \to \Lambda$ transition, the determinations (11a)-(11c) for F, D, and γ from Eqs. (9a)-(9c) are subject to correspondingly larger uncertainties. In particular, the value of F/D deduced from Eqs. (11a) and (11b), while consistent with the values given by earlier investigations [18,19] and used in the nucleon spin-structure analyses [20] [assuming exact SU(3) symmetry], has an uncertainty which is an order of magnitude larger than the error quoted by the cited authors. Although our calculations do not fully support their optimistic estimation of error, uncertainties of F and D smaller than those quoted in Eqs. (11a) and (11b) could be justified as follows.

Eqs. (11) show that errors associated with the quantities F, D, and γ are much larger than those for δ . They arise almost entirely from the poorly determined ratio r_4 , Eq. (10d), of g_A/g_V for $\Xi^- \to \Lambda$ transitions, which does *not* enter the calculation, Eq. (9d), for δ . Using available data, we have shown that the SU(3)-breaking parameter δ differs from zero by less than one standard deviation. The following remarks can be made:

- If we were to accordingly set $\delta = 0$, then only three parameters would remain to be determined. The first impulse, to use the more accurate data for r_1 , r_2 , and r_3 to solve for F, D and γ , turns out to be unfeasible. Eqs. (8a)–(8c) are no longer linearly independent if $\delta = 0$, as shown explicitly in Eq. (9d). Therefore, one must include r_4 to solve for F, D and γ . While it is a matter of choice whether one chooses r_2 or r_3 as the redundant datum, the solution obtained by solving (8a),(8c), and (8d) yields a smaller error for F than if (8a), (8b), and (8d) were used [21]. The first set of results for F, D, and γ is shown, together with a least-squares fit to all four Eqs.(8a)-(8d), in the second column of Table I.
- If the SU(3)-breaking parameter γ is assumed to be similarly small, then an analysis neglecting both γ and δ requires only 2 r-values to determine F and D individually. Using the relatively well-measured g_A/g_V ratios (r_1, r_3) [rather than (r_1, r_2) [21]] reported in Eqs. (10a) and (10c), the solution is listed on the right side of the third column of Table I, and compared to a least-squares fit to all four r-values.
- The linear dependence of Eqs. (8a)-(8c), when δ is assumed to be zero, suggests the following approach to their solution. By combining these equations, we find

$$r_1 - r_3 = 2D + \gamma = 1.607 \pm 0.017 \tag{8e}$$

and

$$r_1 - r_2 = (2D + \gamma)/3 = 0.549 \pm 0.015$$
 (8f)

Defining the left-hand sides as x_3 and x_2 , respectively, we see that the equations require $3x_2$ and x_3 to coincide. In the absence of other information, the most reasonable procedure is to take the weighted mean of $3x_2$ and and x_3 as the best estimate for $2D + \gamma$. Thus, we obtain

$$2D + \gamma = 1.612 \pm 0.016 \tag{8g}$$

Combined with

$$4D + \gamma = 3(r_1 - r_4) = 3.051 \pm 0.150 \tag{9e}$$

which follows from Eqs.(9b) and (9c) when $\delta = 0$, we obtain the solutions

$$D = 0.720 \pm 0.075 \tag{11e}$$

$$\gamma = 0.173 \pm 0.154,\tag{11}$$

which, combined with Eq. (8c), yield

$$F = 0.380 \pm 0.077 \tag{11g}$$

in substantial agreement with the least-squares fit shown in Table I to the data (8a)-(8d) under the assumption that SU(3)-breaking in baryonic beta-transitions can be represented by the sole parameter γ .

Thus, according to our analysis, the "spin crisis" cannot be blamed on the effects of SU(3)-breaking and its resolution must be sought elsewhere. A critical test of our proposed method of analysis requires better data, and application to transitions other than those listed in Eqs.(8). In principle, measurement of g_A/g_V for four different baryonic beta-decays uniquely determines the couplings F and D which survive in the symmetric limit, and two parameters γ and δ representing SU(3)-breaking effects arising from mass-splittings. Measurement of additional beta-transitions would over-determine the fit and provide a test of the theoretical description proposed here to take account of the SU(3)-breaking effects which must arise from mass-splittings. It is probable that these may become available in the foreseeable future [22]. As examples of application to possible future data, we calculate g_A for $\Sigma^{\pm} \to \Lambda$,

$$(-q_A)_{\Sigma^{\pm} \to \Lambda} = D + \delta = 0.722 \pm 0.084,$$
 (12a)

and g_A/g_V for $\Xi^- \to \Sigma^0$,

$$(-g_A/g_V)_{\Xi^- \to \Sigma^0} = F + D = 1.144 \pm 0.012,$$
 (12b)

using the parameters found in Eqs. (11). The equality of the matrix-elements for $\Sigma^{\pm} \to \Lambda$ decays is required by the assumption that they arise from mirror-conjugate components of the same isovector weak current. Similarly, isospin invariance requires the r-value for $\Xi^0 \to \Sigma^+$ to be equal to the one, Eq.(12b), for $\Xi^- \to \Sigma^0$.

After this note had been written, we came across the paper by the KTeV Collaboration [23] reporting the decay $\Xi^0 \to \Sigma^+ e^- \bar{\nu}_e$, which obtained

$$(-g_A/g_V)_{\Xi^0 \to \Sigma^+} = 1.32 \pm 0.20,$$
 (12d)

which is consistent with our expectation (12b).

Note that the value

$$F/D = 0.542 \pm 0.128,\tag{13}$$

deduced from Eqs. (11a) and (11b) is consistent with the representative value: $F/D = 0.575 \pm 0.016$ given in [18,19], and used in the spin-structure analyses [20].

3. Summary

Effects of flavor-SU(3) breaking, arising from quark mass-differences, on hyperon betadecays have been phenomenologically analyzed. A scheme is presented which requires two additional parameters to describe the effects of SU(3) breaking in the allowed approximation. Application to the available data indicates that such effects have not yet been clearly established, and therefore the values of F and D deduced from earlier analyses, ignoring SU(3)-breaking effects, do not require any drastic revision. One of these parameters δ is found to be consistent with zero, within one standard deviation, at a level of accuracy comparable to the g_A/g_V determinations. Test of the parametrization proposed in this paper, and demonstration of SU(3)-breaking effects in hyperon beta-decays, requires better data, and also for transitions beyond those taken into account in our numerical analysis.

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TABLES

TABLE I. Parameters describing baryonic beta-transitions, with and without $\mathrm{SU}(3)$ symmetry-breaking

Parameters	SU(3)-breaking (I)	SU(3)-breaking (II)	SU(3) symmetry	
	$(\gamma \neq 0, \delta \neq 0)$	$(\gamma \neq 0, \delta = 0)$	$(\gamma = \delta = 0)$	
		$[\chi^2$ -fit to $r_{1-4}]$ $[r_1, r_3, r_4]$	$\left[\chi^2\text{-fit to }r_{1-4}\right] \left[r_1, r_2 \text{ only}\right]$	
F	0.402 ± 0.085	$0.374 \pm 0.075 0.382 \pm 0.078$	0.463 ± 0.025 0.444 ± 0.023	
D	0.742 ± 0.080	$0.719 \pm 0.075 \ \ 0.722 \pm 0.080$	0.804 ± 0.025 0.823 ± 0.023	
γ	0.123 ± 0.164	$0.173 \pm 0.150 \ \ 0.163 \pm 0.154$	0 0	
δ	-0.020 ± 0.024	0 0	0 0	
F/D	0.542 ± 0.128	$0.520 \pm 0.117 \ \ 0.529 \pm 0.123$	0.576 ± 0.036 0.539 ± 0.031	

TABLE II. Calculated value of \boldsymbol{r}_j under various assumptions.

Transitions	SU(3)-breaking (I)	SU(3)-breaking (II)		SU(3) symmetry	
	$(\gamma \neq 0, \delta \neq 0)$	$(\gamma \neq 0, \delta = 0)$		$(\gamma = \delta = 0)$	
		[χ^2 -fit to r_{1-4}	$[r_1, r_3, r_4]$	$[\chi^2$ -fit to $r_{1-4}]$	$[r_1, r_2 \text{ only}]$
$(-g_A/g_V)_{n\to p}$	input	1.2667 ± 0.183	37 input	1.2670 ± 0.0350	input
$(-g_A/g_V)_{\Lambda \to p}$	"	0.729 ± 0.146	0.731 ± 0.131	0.731 ± 0.026	"
$(-g_A/g_V)_{\Sigma^-\to n}$	"	-0.345 ± 0.10	6 input	-0.341 ± 0.035 -	-0.379 ± 0.132
$(-g_A/g_V)_{\Xi^-\to\Lambda}$	"	0.25 ± 0.15	"	0.20 ± 0.03	0.17 ± 0.02
χ^2	-	0.68	0.79	1.96	8.09
$(-g_A/g_V)_{\Xi^0\to\Sigma^+}$	1.144 ± 0.012	1.093 ± 0.106	1.104 ± 0.112	1.267 ± 0.035	1.267 ± 0.033
$(-g_A)_{\Sigma^{\pm}\to\Lambda}$	0.722 ± 0.084	0.719 ± 0.075	0.722 ± 0.080	0.804 ± 0.025	0.823 ± 0.023